X. *On the Thermal Conductivity of Water.*

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THE experiments on the thermal conductivity of water, of which an account is given in the following paper, were undertaken at the wish of Sir WILLIAM THOMSON, and by a method devised by him some years ago. They have been carried out in several successive Winter Sessions in the Physical Laboratory of the University of Glasgow, with the assistance of students and experimental scholars, among whom I must mention specially Mr. J. REID, Jun., and Mr. M. T. BROWN.

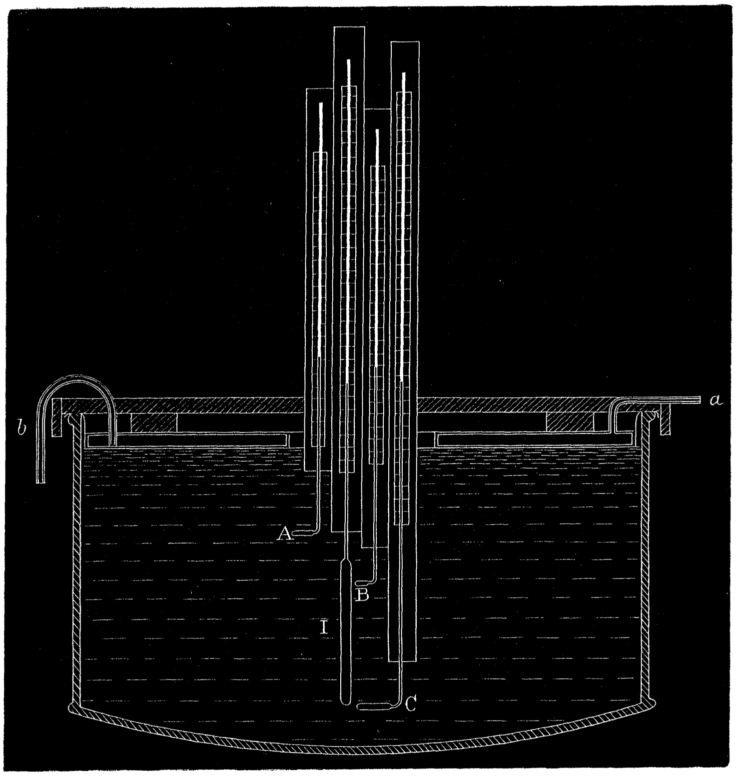
Description of apparatus.—Figs. 1 and 2 show the arrangement of apparatus for experiments on the thermal conductivity of a liquid. In each case I have a vessel or tank so large that the heat lost at the sides does not affect sensibly the condition of the central portion of the liquid, either directly, or by convection currents set up close to the sides. The water or other liquid to be experimented on is contained in this vessel and is heated from above.

The tank, fig. 1, is a cylindrical vessel of thin sheet iron. It is 120 centims. in diameter and 60 centims. high. At the top there is a hollow cover in contact with the water in the tank, and a current of steam is kept blowing through this hollow cover. The steam enters by the pipe *a* and is blown out by the pipe *b*, which is made to dip down to the bottom of the steam chamber in such a way that the water formed in the chamber by the condensation of the steam is blown out through this pipe along with the surplus steam. In the middle of the hollow cover, or steam chamber, there is a rectangular opening, walled in so as to make the chamber steam-tight, and the stems of the thermometers employed in the experiments pass up through this opening. The water being heated from above transmits the heat downward to the layers beneath.

The tank shown in fig. 2 is a wooden vessel, an ordinary wine cask, about 64 centims. in diameter, and 90 centims. high. It had the disadvantage of not being uniform in section from top to bottom, but this had been done away with in tanks recently constructed. I require a *wooden* vessel for experimenting on solutions of salts in water. Solution of sulphate of zinc, for example, quickly corrodes and eats its way through an iron vessel; and solution of sulphate of copper (at any rate solution of the sulphate of copper of commerce) eats through both iron and copper.

The mode of applying heat in this second apparatus is different from that described above. When an experiment is to be made the tank is filled with water (or other liquid) up to the level, D, and after the water has been carefully stirred from top to bottom and has come to rest after the stirring, a "floater" is placed on the surface. The floater is a large thin board with its edges carefully bevelled all round. It covers nearly the whole surface of the liquid below. Hot water is allowed to flow, slowly at first, but afterwards as quickly as is convenient, on the top of the floater, and a thick layer of very hot water is laid on the top of the colder water beneath. The edges of the floater are bevelled so that the hot water falling on it is shot off horizontally at the edges without any downward motion, and therefore with no tendency to mingle with the colder water, of greater density, below; and if the operation of laying on a layer of hot water is carefully carried out, there is scarcely any mixing of the hot water with the cold. This can be seen if the experiment is tried in a large glass vessel. A layer of hot water can easily be deposited on the top of a quantity of cold water, with so little mixing that the layer of separation intermediate between the hot and the cold, which are distinguishable by the difference in refractive indices of hot and cold water, is *seen* to be not thicker than a sheet of paper. I have also tried experiments as to mixing on a large scale in the sheet-iron tank, fig. 1. On running

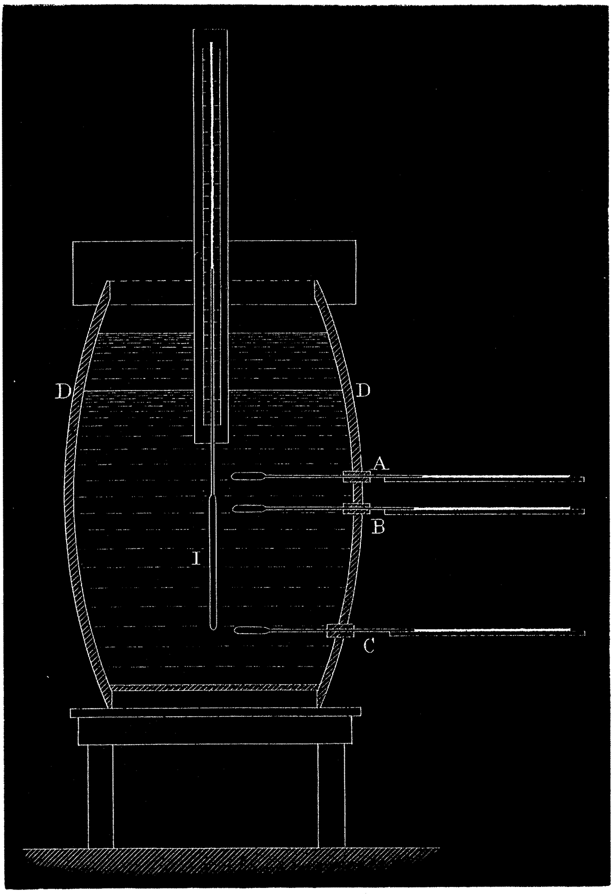
Fig. 1.



a layer of hot water on the top of a quantity of cold with the aid of a properly constructed floater, and applying the finger outside at the place of separation, a space of separation of the hot from the colder water can, even in this rough way, be ascertained to be certainly not so thick as half a centimetre.

Figs. 1 and 2 show also the arrangement of the thermometers employed. In each case four are used. Of these three have their bulbs horizontal. Each of these shows the temperature of the layer in which it is placed. The thermometers A and B in the diagrams show the difference of temperature between the two sides of a stratum of the liquid a few centimetres (5 to 10 centims.) thick. The thermometer C is merely used to show that heat has been conducted down to the bottom of the portion of liquid under experiment, and is therefore being conducted out below. When the thermometer C begins to rise the experiment is at an end.

Fig. 2.



The fourth thermometer, marked I in the diagrams, is called the *integrating thermometer*. It has a bulb 30 centims. long, which is placed vertically in the liquid. This thermometer is intended to show the whole quantity of heat that passes during any time into the stratum of the liquid bounded by horizontal planes through the top and bottom of its bulb.

When heat is being conducted downward through the liquid, the various parts of the integrating thermometer are at different temperatures, the temperatures decreasing from above downwards. On the whole, however, the temperature indicated by the integrating thermometer is the *average* temperature from top to bottom of the layer of liquid which contains the bulb of the thermometer.

I have verified by experiments that the integrating thermometer does give with very considerable accuracy the average temperature from top to bottom of a cylindrical layer of liquid in which it is contained. For this purpose I placed the thermometer vertical in a cylindrical vessel, the bottom of the bulb being at the bottom of the vessel. The bottom of the vessel was plane and was horizontal. I then poured in an inch or two of cold water. On the top of that layer I deposited a layer of warmer water, then a layer of still warmer water, and so on, making a succession of layers of water of gradually increasing temperature till the level of the top of the bulb was reached. I then read off the temperature indicated by the thermometer as the average of the whole. I next stirred the whole from top to bottom so as to equalize the temperature throughout and read the thermometer again. Many such experiments were tried, and in every case I found that the alteration in the reading was very small, and was certainly not more than variation which could be accounted for by unavoidable errors introduced by changes of temperature of the parts of the containing vessel and of the stirrer.* These tests were so severe in comparison with the requirements of the present experiments on the conductivity of heat by liquids, that I am satisfied that no error of consequence, in comparison with uncertainties that I have not yet been able to get rid of, is introduced by want of *perfect* averaging by the integrating thermometer.

From the description of the arrangements for these experiments, it will be seen that we have a layer of liquid, of known thickness, between the thermometers A and B kept with one of its sides hotter than the other. The thermometers A and B give the temperatures of the two sides of the layer. The heat conducted across it warms the liquid beneath, and the whole heat that flows across the layer between A and B in any given time is known from the readings of the integrating thermometer at the beginning and end of the interval. Now if the temperature of the layers at A and B could be kept constant during an experiment the calculation of the thermal conductivity of the liquid from these data would be of the simplest possible kind.

The quantity of heat that passes across the stratum between A and B in any time is in simple proportion to the difference of temperatures between the two sides of the stratum; it is in simple proportion to the time; it is in simple proportion to the area of the stratum; it is inversely proportional to the cross-sectional *area* of the stratum;

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* It is hardly necessary to say that alteration of specific heat of the water by change of temperature could not be taken into account in considering the results of experiments such as those described.

and it is in simple proportion to the thermal conductivity of the liquid. Now let T and T′ denote the temperatures of A and B respectively—supposed for the moment constant—let a denote the thickness of the layer between A and B, let t denote the time in seconds, k the conductivity, and Q the quantity of heat conducted across the stratum whose area is A ;—then we should have

$$Q=k\frac{T-T'}{a}At \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, taking as unit quantity of heat the quantity required to raise unit bulk of water one degree, and neglecting for the present differences of specific heat, in the different layers of the liquid column, due to differences of temperature, we have, if l be the length of the integrating thermometer, D the difference between the average temperature from top to bottom at the beginning and the average temperature from top to bottom at the end, and if c be the specific heat of the liquid per unit of bulk

$$Q=A.l.c.D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, equating (1) and (2), we have

$$k\frac{T-T'}{a}.A.t=A.l.c.D$$

or

$$\frac{k}{c}=\frac{Dl}{\frac{T-T'}{a}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$k=\frac{Dlc}{\frac{T-T'}{a}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In calculating from the results of experiments on the conductivity of water I have taken its specific heat as being equal to 1. The experiments that I have made up to the present time do not permit of my being able to take into account either variations of specific heat in the different parts of the layer tested, or variations of the average specific heat in the layer from top to bottom from that of water at the temperature chosen for defining unit specific heat. The formula, therefore, which would be used for calculation, if the conditions mentioned above—namely, the constancy during an interval of time of the temperature at the levels A and B—were fulfilled, is

$$k=\frac{Dl}{\frac{T-T'}{a}t}.$$

It will be seen, however, from the method of making the experiments that to fulfil this condition was impossible.

In both of the modes of conducting the experiments the heating arrangement is at a considerable distance from the layers which the thermometers A and B represent; and the heat supplied travels gradually downward.

In the arrangement (fig. 1) the rise of the two thermometers A and B goes on continuously. In the arrangement of fig. 2, where there was only a limited supply of heat, namely, that belonging to the layer of hot water poured on the top, both thermometers first gradually rise in temperature and then gradually fall.

The following, then, was the mode of experimenting and of calculation. The water to be experimented on was first carefully stirred from top to bottom to equalize the temperature throughout. The heating was applied and the temperatures of all four thermometers were read. An hour or so afterwards the regular observations began. After that time the thermometer A was generally found to be commencing to rise slowly. Somewhat later the thermometers I and B were found to be rising, and from that time until the lowest thermometer was observed to be rising the reading of the thermometers was carried on continuously at intervals of five minutes or of ten minutes. The observations are generally carried on for five or six hours and are therefore laborious.

It is unnecessary to put down here the readings taken in the various experiments carried out. It is only a small portion of the series of numbers recorded in any one experiment that is made use of for calculation. The part that is made use of is that which keeps the difference of temperatures of the thermometers A and B as nearly as possible constant for a considerable time. In the first mentioned method of heating it is that part at which the two thermometers A and B are both rising at nearly the same rate. In the second method it is that part of the time of observation during which the two thermometers remain most nearly stationary. During this part of the time the increase in temperature of the integrating thermometer is taken. The *average* temperature indicated by A is taken as the temperature of the level which is represented by it, and the *average* temperature of the thermometer B is taken as the temperature of the level which is represented by it. These are used in the formula given above as the temperatures T and T′ respectively.

A specimen of part of a series of thermometer readings, extending over two hours, is given in Table I. appended. The heating was carried on by the method of fig. 1. The heating was commenced at about 9 A.M., and the observations were carried on till 4 P.M., when they were discontinued as the thermometer at the bottom of the stratum occupied by the integrator began to rise very sensibly.

Table II. contains results calculated from a number of satisfactory experiments. From these and from the consideration of other results, I conclude that the conductivity for heat of water at the temperature at which I have experimented is to be reckoned at ·0023 square centims. per second.

The temperatures at which the experiments were made have varied from 12° C. to 17° C. I have not yet made any attempt to trace difference in conductivity corresponding with different temperatures of the water.

I have also commenced experimenting on the conductivity for heat of saline solutions, but I am not yet prepared to give numerical results that I should consider satisfactory. In order to make a comparison between the conductivity of water, and that of a liquid other than water, it is necessary to know the specific heat of the liquid, or, in the case of a saline solution, of the saline solution at the density which it has in the experiment on conductivity. Solution of sulphate of zinc has been experimented on in a preliminary way for conductivity, and its specific heat at various densities has been determined. I hope, however, with improved apparatus furnished by means of a grant from the Government Fund of £4,000, to be able soon to give much better results than I can offer at present.

TABLE I.—Conduction of heat by water. Series of thermometer readings taken with arrangement of fig. 1, December 10, 1873. Time, 1.30 P.M., to 3.30 P.M. Thermometers A, B, I, C, as shown in fig. 1.

Time.	THERMOMETERS.			
	A.	B.	I.	C.
1·30	140·5	113·05	38·95	61·7
·35	143·1	113·8	39·35	
·40	145·0	114·9	40·15	
·45	147·2	115·75	40·8	
·50	148·45	116·5	41·05	
·55	149·45	116·5	41·15	
2·0	151·6	117·15	41·9	
·5	152·95	118·15	42·3	
·10	154·45	118·85	42·65	
·15	157·95	119·85	43·05	
·20	159·95	120·5	43·75	
·25	161·95	121·5	44·05	
·30	163·4	122·0	44·6	61·75
·35	166·3	122·6	45·1	
·40	168·0	123·6	45·9	
·45	169·3	124·3	46·25	
·50	170·0	124·65	46·6	
·55	170·1	124·85	46·65	
3·0	170·95	125·45	47·05	
·5	174·0	126·85	47·95	
·10	176·05	128·0	48·5	
·15	178·2	129·0	49·15	
·20	180·3	129·65	49·95	
·25	181·5	130·2	50·05	
·30	182·6	130·85	50·6	62·05

TABLE II.—Results of experiments on thermal conductivity of water, expressed in square centimetres per second.

	Date and hour of experiment.	Results.	Remarks. Heating commenced always about 9 A.M.
1	1873, Dec. 3, 1 P.M. to 2 P.M. . . .	·00209	Results 1–13 obtained by method of fig. 1.
2	" " 9, 10.15 A.M. to 11.15 A.M.	·00276	Probably too high a result owing to some disturbance at the beginning of the experiment.
3	" " " 11.20 A.M. to 12.20 P.M.	·00226	
4	" " " 12.25 P.M. to 1.25 P.M. .	·00212	
5	" " " 1.30 P.M. to 2.30 P.M. .	·00230	
6	. . . .	·00235	Other results calculated from observations of December 9th taking different intervals of time during the experiment.
7	. . . .	·00230	
8	. . . .	·00236	
9	. . . .	·00228	
10	1873, Dec. 10, 11 A.M. to 12 NOON . .	·00244	
11	" " " 12 NOON to 1 P.M. . .	·00218	
12	" " " 1.30 P.M. to 2.30 P.M. .	·00229	
13	" " " 2.30 P.M. to 3.30 P.M. .	·00231	
14	1876, March 24, 2 P.M. to 3 P.M. . .	·00197	Results 14–16 obtained by method of fig. 2. The results given by this method have uniformly given a smaller number for the conductivity than that given by the first method. I attribute this to one or both of two causes. First, the supply of heat is limited and is not large enough in the arrangements that I had at my command when these experiments were made, and, secondly, the vessel in which the experiments were made was not, I believe, large enough to make sure of no loss by the sides. Some experiments that I made in a much smaller vessel some time ago pointed clearly to this cause, giving me a very low number for the value of *k*. I hope with improved apparatus to make sure of the result; but in the meantime I believe it to be about 0·002 of a square centimetre per second.
15	" " " 3 P.M. to 4 P.M. . .	·00212	
16	" " 31, 1.40 P.M. to 2.40 P.M.	·00194	

ADDITIONAL NOTE.

(Added December, 1880.)

In the foregoing paper I have supposed that by taking a vessel of very large horizontal cross section I should be able to eliminate the sensible effect of conduction of heat downwards by the sides of the vessel. However, considering the importance of the part which the sides might play, and the statements in books and published writings that the conduction of heat downwards in liquids, observed by DEPRETZ and others, was wholly, or nearly wholly, due to conduction by the sides of the containing vessel, it was considered advisable to put the matter to direct test. The following experiments were accordingly made at the suggestion of Professor STOKES in the months of March and April last.

In order to test any possible effect producible by the conduction of heat downwards by the sides of the vessel and by the convection currents thereby established, the following arrangement was made. Within the cylindric tank in which the testing was carried on, a circular screen of thin cotton material was hung, which was everywhere 6½ centims. from the outer walls, and which extended from top to bottom of the tank. Any convective action due to conduction of heat downwards by the walls of the vessel would go on outside the screen: and the enclosed column, 77 centims. in diameter, would be but little affected by these slight motions. The thermometers projected into the middle of the enclosed column through small holes cut in the screen.

Experiments made in March and April, 1880, with and without the screen, gave the following numbers for the apparent thermal conductivity of the water (in terms of an undetermined unit):—

without screen	·00137,	·00135,	·00130;
with screen	·00154,	·00149,	·00158.

These numbers, though not in complete agreement, are yet quite of the same order. They seem to indicate that the screen does produce some effect, and I propose to carry out a much more complete series of experiments in order to determine exactly to what this is due. One of the greatest difficulties of the investigation is the determination of what is to be considered as the effective distance between the levels represented by the thermometers A and B in the diagram given in the foregoing paper; and it prevented me from reducing the results of these last experiments satisfactorily to absolute measure, though it does not at all affect their *relative* values. Any error in estimating this distance produces a proportionate error in the absolute value of the numerical result. I am therefore making preparations to eliminate as far as I can this source of error. I believe however that the numbers given in Table II. may be taken as approximations to the absolute conductivity of water, which has not been hitherto even roughly determined.